Labor Market Dynamism and Job Polarization*

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Abstract

Over the last two decades labor market dynamism, measured by flows of workers between employers, declined substantially in the US. During the same period employment polarized into low and high-skill jobs. This paper shows that the two trends are linked. First, I provide a framework to study employment and worker flows in presence of two-sided heterogeneity. I analyze within this framework the effects of routine-biased technological change and the increasing supply of college graduates on labor market flows. When routine-biased technological change displaces mid-skill jobs, it lowers the opportunity to move up to better jobs for low-skilled workers. Similarly, highly skilled workers have less opportunity to take typical stepping stone jobs and either trade down to lower-skill jobs or start employment higher up the job ladder. The rising share of college graduates puts further pressure on labor markets by increasing competition for jobs from top to bottom. In equilibrium, workers trade down to jobs with lower skill intensity to gain employment but may find it harder to move up as they are competing with more highly educated workers. I quantitatively assess whether such mechanisms contributed to the fall in labor market dynamism, by estimating the model using data on labor market flows. I find that routine-biased technological change accounts for about one-third of the decline in job-to-job mobility for workers without a college degree, while the remaining decline in mobility is mainly driven by a decline in the dispersion of match-specific productivity and its innovation rate.

Keywords: Job Polarization, Sorting, On-the-job Search, Skill Distributions, Job Competition
JEL Codes: E24, J62, J64, O33

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1 Introduction

In the last two decades the US labor market experienced a decline in labor market mobility and a continued polarization of employment, that is a shift in employment away from mid-skill jobs towards low and high-skill jobs. While technology has been identified as an important driver of polarization in employment, its impact on the sorting of workers to jobs and the underlying causes for the decline in labor market mobility, as measured by job-finding rates on the job, are less clear.[1]

First, I document that the polarization in overall employment is not present conditional on education level, but instead, a shift towards lower-skill jobs is observed for workers of all education levels. The two results are nevertheless consistent, as the share of college graduates in the workforce has grown substantially. Further, I show that the shifts in employment are closely mirrored in job-finding rates by education and job type. In contrast to job-finding rates from unemployment, job-to-job transition rates have been falling on average. I argue that the displacement of mid-skill jobs and the increasing supply of college graduates may explain the decline in job-to-job transition rates. The displacement of mid-skill jobs leaves workers with lower education levels with less opportunity to move up the job ladder as high-skill jobs are mostly out of reach, and even more so with an increasing share of college graduates in the workforce. Thus, they also move between jobs. High-skilled workers are less likely to find a stepping-stone job in the middle of skill distribution and start employment directly in higher-skill jobs or trade down to lower-skill jobs. Thus, the impact on job-to-job transition rates is ambiguous for higher-skill workers. Further, the increased demand for high-skill jobs may increase higher-educated workers' job-to-job transition rates directly. The changes in the composition of labor demand have been accompanied by a large increase in the number of college-educated workers. Additional highly educated workers intensify competition for highly skilled jobs and in response workers trade down to lower skill jobs, that is competition trickles down the job ladder and intensifies at all types of jobs.

To analyze the observed labor market trends further and to quantify the importance of the different mechanisms for both sorting and worker mobility, I build a novel framework that links the allocation of employment across jobs with worker mobility. The model embeds production with heterogeneous occupations and workers into a directed search model of the labor market. The setup highlights that, when workers have a comparative advantage in some jobs, the division of production into occupations will depend both upon the relative productivity of occupations and the supply of skills. Furthermore, as workers compete with each other for jobs, the incentives for job search depend not only upon the value of employment but also on the composition of the pool of applicants. Thus, the allocation of workers to jobs, their mobility between jobs, and overall employment are determined jointly in equilibrium.

[1] At the same time, the decline in worker mobility raised concerns about the limited opportunities workers have to move to better jobs. See Moscarini and Postel-Vinay (2016) and Abel, Florida, and Gabe (2018) for evidence of a "failing" job ladder in recent years. Closely related, there is also concern about whether college graduates are increasingly employed in jobs that do not require a college degree, see for example Abel and Deitz (2014) for a discussion of the employment of college graduates in recent years.

To capture the key characteristics of the labor market the model incorporates search frictions, on-the-job search, two-sided heterogeneity, and sorting. These features are essential to study labor market flows in the presence of rich heterogeneity and assortative matching, as observed in the data. The allocation of workers to jobs is not random, for example, college graduates are more likely to work as managers, while high school graduates are more likely to work as waiters. And productivity differences across occupations persist because their outputs are imperfect substitutes. The proposed framework is consistent with the observed sorting and mobility patterns.

Then, I proceed by applying the framework to study the recent experiences in the US labor market. Within the framework, I provide a quantitative assessment of the importance of changes in labor demand, rising education levels, and the search process for the observed changes in employment, sorting, and the decline in the job-to-job transition rate. To do so I estimate the main model parameters separately for 1997 and 2017 using mainly moments on labor market flow rates based on CPS microdata. For the estimation, I group jobs based on two criteria: (1) whether the job's tasks are predominantly routine and (2) whether the job has mainly cognitive or manual skill requirements. Furthermore, I group workers based on their education level as a proxy for their skill level. The model captures well the observed distribution of job-finding rates by education-occupation group and the pattern of job-to-job transitions across jobs. The parameter estimates reflect large shifts in the composition of labor demand and labor supply. Further, the model captures flexibly additional changes in the hiring process. I use the model to show which of these changes are driving the observed changes in employment, sorting, and job-to-job transition rates. A shift solely in labor demand can account for the overall employment shift across jobs. However, by itself, routine-biased technological change can not account for the shifts in employment by education and occupation groups. It is essential to incorporate the rising education levels of workers to account for the common downward shift in employment towards lower-skill jobs for workers of all education levels. Further, the changes in labor demand and supply can explain about one-third of the decline in job-to-job transition rates for workers without a college degree, but predict a rise in job-to-job transition rates for college graduates. Finally, the remaining changes in job-to-job transition rates are almost entirely driven by a decline in the dispersion of match-specific productivity and a decline in the arrival rate of shocks to match-specific productivity.

Relation to Literature. First, this article builds upon and contributes to the literature on the recent decline of labor market mobility. Davis and Haltiwanger (2014) and Hyatt and Spletzer (2013) provide empirical evidence for a decline in labor market mobility and argue that while composition shifts in the labor force are important, they can only explain 30-40% of the decline in mobility. Furthermore, they provide evidence that shifts in employment across industries has not been a driver of the decline as workers reallocate towards industries with traditionally higher turnover. In this study, I build upon their evidence, but focus upon a novel explanation of the decline in mobility. That is, changes in composition of the supply of and demand for skills have far-reaching equilibrium effects on labor markets.

Cairo (2013) studies the effect of increasing training costs on turnover in a random search model with large firms. She finds that increasing training costs, acting as a fixed cost to hiring that is subsequently lost when separating, decreases turnover. By increasing the cost of match formation the willingness to sustain matches under bad conditions increases and thus turnover declines. Fujita (2018) argues that increasing "turbulence" - a higher rate of skill loss at separation from employment - can explain lower turnover. The logic behind his finding is very similar to Cairo (2013), but instead of an increase in the fixed cost of hiring there is an increase in the cost of separation. Both papers argue that their findings can explain a joint decline in job-finding and separation rates. In the descriptive analysis of labor market flows, however, I find that separations to non-employment conditional on a workers education level are increasing while job finding rates decline over the last two decades. This paper contributes to the findings of those papers by analyzing worker mobility in a framework with sorting and on-the-job search, two essential features of the data, and providing a rationale for declining worker mobility in the absence of changes in matching and separation of costs.

Engbom (2017) highlights aging and its interaction with firms hiring decisions and innovation as a force driving down labor demand and turnover. Mercan (2018) argues that the availability of information about workers has increased and thus allows tighter selection at the hiring stage, leading to fewer job-to-job moves. While these papers address potential explanations for the decline in mobility and employment, they do not address the sorting of workers to jobs and whether the decline in mobility is related to changes in sorting patterns. One exception is recent work by Eeckhout and Weng (2020) who study mobility and sorting. They focus on changes in the complementarity between workers' unobserved skills and jobs technology, but I focus on changes in demand for and supply of skills. Pries and Rogerson (2022) document using a different data source, that a share of the decline in turnover can be explained by a decline in short duration matches and argue with a search model that an improved screening process can account for this finding. The findings in the current paper will be consistent with those results. While these papers study closely related questions they focus on different mechanisms and the importance of each mechanism for the decline in mobility is still an open question. Thus I consider them complementary to this paper. The main contribution of my paper is to analyze worker mobility and shifting employment in a setting where there is not only sorting, but also competition between workers leading to rich equilibrium interaction between worker mobility and the demand for and supply of skills.

Second, this study also contributes to the literature on models with search frictions and sorting in the labor market. Barnichon and Zylberberg (2018) consider a setup of the labor market with similar features as in this paper and analyze employment by education level of workers over the business cycle. They find that highly-educated workers are downgrading towards low-skill jobs in downturns, which leads to more unemployment for workers with less education as high-skilled workers are preferentially hired. This paper is based on a similar job competition mechanism and they provide outside evidence that the mechanism is relevant for the allocation of workers to jobs. Though related, they do not focus on the trend in worker mobility and its possible causes. Furthermore, they do not include on-the-job search, which is at the core of this paper. Lise and Robin (2017) also study sorting over the business cycle, but use a random search framework that, in contrast, does not feature explicit competition at the hiring stage. While, they address only business cycles and I focus on trend changes in the labor market, it is also the key mechanisms of how sorting happens in the labor market that are different. I focus on competition between applicants and directed search, while in their framework sorting is entirely based on matching sets. By allowing for competition between workers at the hiring stage, I can address to what extent high skilled workers crowd out lower skilled workers from particular jobs and employment.

Third, the current article is also closely related to the literature on technological change, job polarization and wage inequality. Following the contributions by Goos and Manning (2003) and Autor, Levy, and Murnane (2003) a large literature has analyzed how technology can explain job polarization and other labor market outcomes, for instance Acemoglu and Autor (2011), Goos, Manning, and Salomons (2014) and Stokey (2018). Cortes, Jaimovich, and Siu (2017) build on this literature and study a frictionless model of the labor market to analyze to what extent the declining labor force participation rate can be explained by technological factors. In this paper I proceed in a similar manner, but focus instead on the role of technology for job search both on and off the job and its implications for the sorting of workers to jobs. Beaudry, Green, and Sand (2016) and Aum (2017) provide evidence that the supply of educated workers outpaced the demand for skilled workers since 2000. In this paper I find a similar pattern and will take into account both shifts in demand for jobs and the supply of educated workers.

The remainder of the article is organized as follows. Section 2 provides a descriptive overview of the recent trends in worker mobility and employment. In Section 3 I lay out the theoretical framework. The structural estimation setup follows in Section 4, where I discuss identification and present the estimated parameters and model fit. In Section 5 I perform the decomposition of the decline in labor market flows using the estimated model. The last section offers concluding remarks.

2 Descriptive Evidence

Data Sources and Sample Selection

The CPS Basic Monthly and Tenure Supplement files for the period 1994 to 2018 are the main data source. The raw data are provided by Sarah, King, Rodgers, Ruggles, and Warren (2018). Occupations are categorized based on their cognitive requirements and routine task content following Autor et al. (2003), see table 1 for an overview. The grouping into routine vs. non-routine jobs captures whether occupations are exposed to displacement by automation technology. The differentiation along cognitive skill requirements categorizes jobs into groups with low vs. high cognitive ability requirements. Workers are differentiated by education levels, which serve as a proxy for general cognitive skills. In the main analysis I use three groups for education levels: (1) at most a high school degree (2) some college, but not a full four year degree and (3) a four year college degree or more. In order to exclude individuals in education and close to retirement, I restrict the sample to individuals of age 25-45. All calculations use CPS sample weights. Further, I restrict the CPS sample to respondents who respond to the survey themselves, to alleviate concerns that changes in the U.S. Census Bureau Respondent Identification Policy affect the measured rate of transitions in the data (Fujita, Moscarini, and Postel-Vinay, 2021).

Table 1: Occupation Groups by Tasks

Tasks	Census Occupations
Non-routine Cognitive	Management Business and financial operations Computer, Engineering and Science Education, Legal, Community Service, Arts and Media Occupations Healthcare Practitioners and Technical Occupations
Routine Cognitive	Sales and Related Office and Administrative Support
Routine Manual	Construction and Extraction Installation, Maintenance and Repair Production Transportation and Material Moving
Non-routine Manual	Service Occupations

See Cortes, Jaimovich, Nekarda, and Siu (2020) for details on classification and mapping to Census Occupation codes.

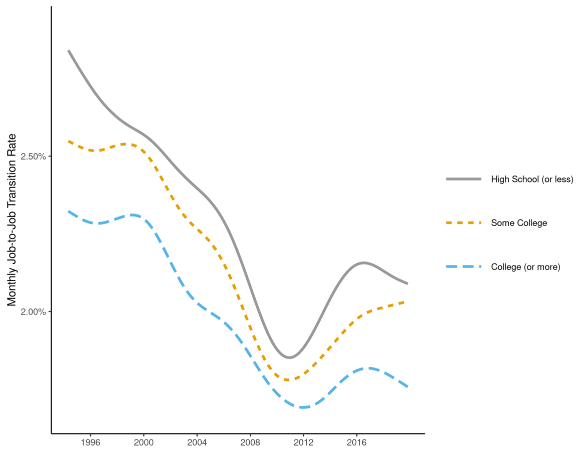
Worker Mobility and Job Polarization

In this section I present evidence of a trend decline in worker mobility, job polarization and the changes in the direction of worker flows across jobs. Over the last two decades there was a substantial decline in job finding rates on the job, while the probability to find a job from unemployment has not changed substantially.

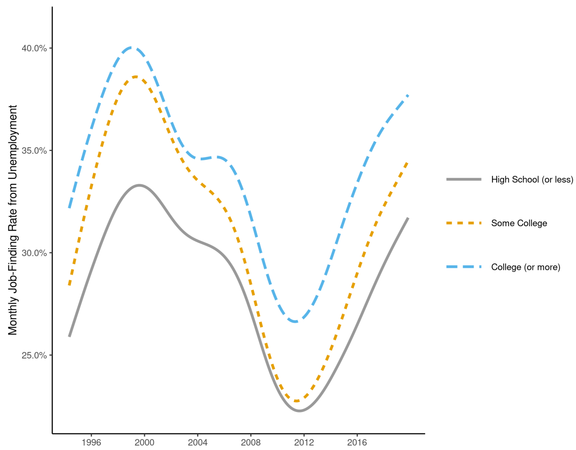
Figure 1 shows in panel (a) the job-to-job transition rate and in panel (b) the job finding rate from unemployment. The job-to-job transition rate declined by over 20% between 1997 and 2017 for workers of all education levels. The decline in the switching rate between jobs has been remarkably common between workers of different education levels, which points towards broad based changes in the labor market. The job finding rate out of unemployment has after the Great Recession recovered to similar levels as were present in the late 90s. Again, the behavior over time is common for workers of different education levels.

Over the same time period there have been broad changes in composition of labor demand and labor supply. Particularly, here I document that employment shifted away from mid skill (routine) employment towards low and high skill (non-routine) jobs. This trend has been called job polarization

Figure 1: Job Finding Rates



(a) Job-to-Job Transition Rate. Own calculations using CPS Basic Monthly Files. Trend calculated from seasonally adjusted monthly transition rates using HP Filter with smoothing parameter 129600.



(b) Unemployment-Employment Transition Rate. Own calculations using CPS Basic Monthly Files. Trend calculated from seasonally adjusted monthly transition rates using HP Filter with smoothing parameter 129600.

and a large literature following the contributions of Autor et al. (2003) and Goos and Manning (2003) has argued that routine biased technological change is behind such changes, but also that trade and off-shoring are other potential causes (Autor, Dorn, and Hanson, 2016, Blinder and Krueger, 2013). I do not focus on the specific causes for changes in the composition of labor demand across jobs, but on its impacts on workers job-finding rates and will henceforth not distinguish those different mechanisms. Parallel, to those changes in the composition of labor demand there has been a substantial increase in the share of college educated workers. Given those parallel changes in the composition of labor demand and labor supply, I present now the changes in employment and job finding rates split by occupation category in the aggregate and conditional on workers education level.

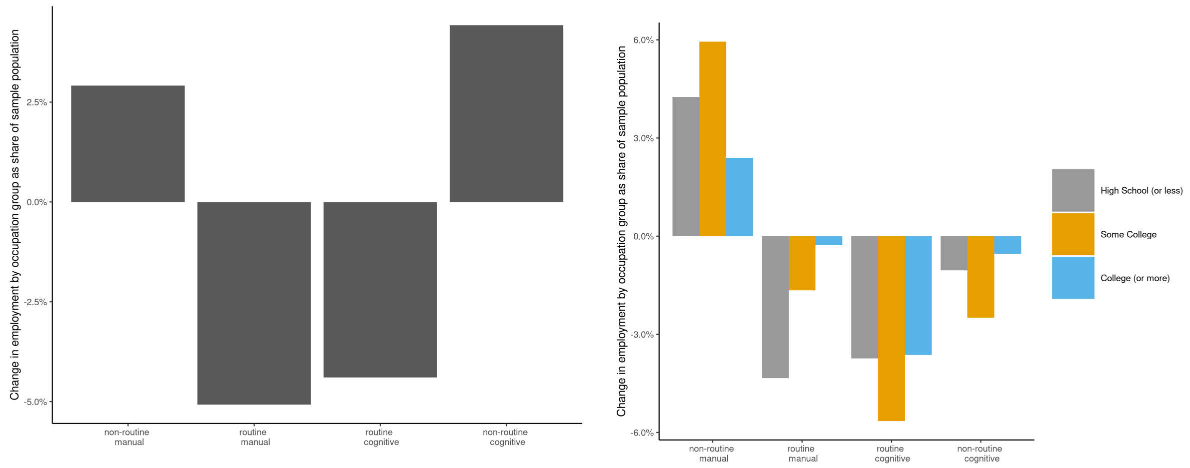
In Figure 2a I show the change in employment per capita between 1997 and 2017 for each occupation group, as defined in table 1. Employment rose in non-routine jobs, while employment in routine jobs declined. The rise in non-routine employment took place both at the bottom and top of the wage distribution, while the decline in routine employment is situated in the middle of the wage distribution[2]. This trend has been termed "Job Polarization" by Goos and Manning (2003). In Figure 2b the change in employment by occupation group is shown again, but conditional on a workers education level. This suggests that polarization of employment is only present in the aggregate, but not conditional on education. There is a clearly distinct pattern in the cross-section compared to the aggregate. First, there does not seem to be an increase in employment in non-routine cognitive jobs. This difference is

[2]The relative pay of these occupation groups has been widely documented. See appendix D for the weekly earnings of those occupation groups, calculated using the CPS outgoing rotation group.

Figure 2: Change in Employment per Capita by Job Type: 1997-2017

(a) Aggregate

(b) Conditional on Education



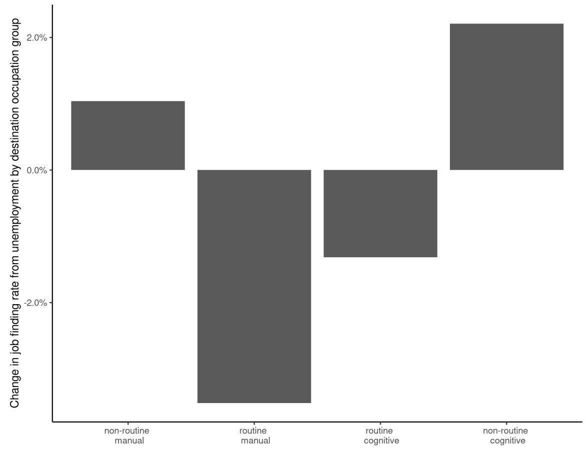
driven by the increase in the supply of college graduates by over 10pp over the same time period, as shown in the appendix in table 9a. Therefore, conditional on a workers education level employment shifts towards low skill jobs. This suggests that the supply of college graduates outpaced demand for high-skill jobs which in turn puts pressure on labor markets from top to bottom. This interpretation is further corroborated by the evidence in Beaudry et al. (2016) and Aum (2017). For the main analysis in the paper I will therefore not only take into account potential changes in the demand for skills, but also in the supply of skills.

The overall shift in composition of employment by job type is closely connected to workers job finding rates, as shown by Cortes et al. (2020). Figure 3 shows the change in the job-finding rate from unemployment by destination occupation group between 1997 and 2017. These changes resemble the overall changes in employment, both in the aggregate and conditional on education. However, as a large share of hires are coming from other employers (Fallick and Fleischman, 2004), I now also document the changes in transitions between employers and split those moves by the occupation group in the hiring job. I find a similar pattern compared to the job-finding rates from unemployment, but the average change has been clearly negative.

One potential explanation for the decline in job-to-job transition rates is that the shift in labor demand has lead to shorter job ladders for workers. While the different types of jobs form a job ladder that workers try to climb, the part of the ladder which is relevant for a worker depends upon her education level. For instance, for workers with at most a high school degree employment is concentrated in non-routine manual and routine jobs. For college educated workers instead employment is concentrated

Figure 3: Change in Job-finding Rate from Unemployment by Destination Job Type: 1997-2017

(a) Aggregate



(b) Conditional on Education

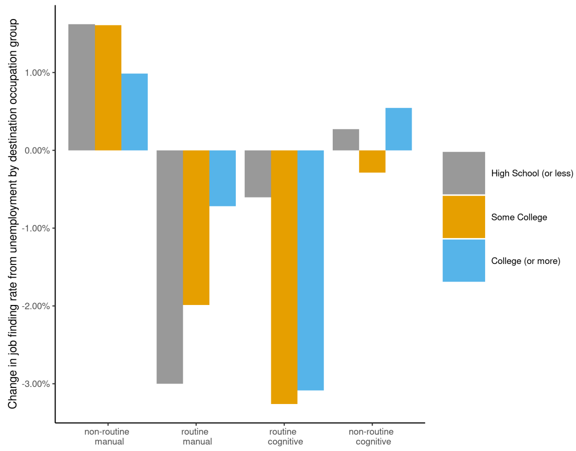
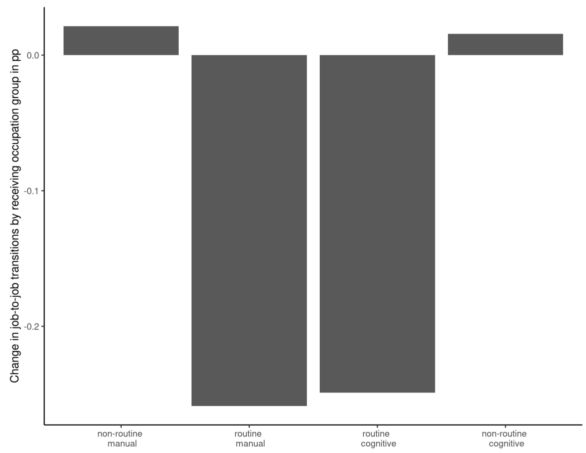
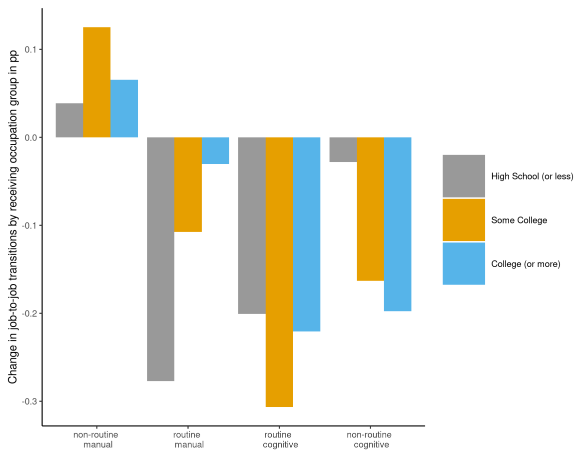


Figure 4: Change in Job-to-job Transition Rate by Destination Job Type: 1997-2017

(a) Aggregate



(b) Conditional on Education



in routine and non-routine cognitive jobs[3]. Thus, as employment shifts away from mid skill jobs it becomes harder for workers with low education levels to move to better jobs, as they have low job finding rates at high skill jobs[4]. They can not easily move to high skill jobs, because they have to compete with college educated workers whose skills are likely more suited for such jobs. Thus, I argue that the opportunity to move up out of low skill employment have diminished for workers with low education

[3]See appendix D for the distribution of employment by job type and education level of workers

[4]The differences in job-finding rates from non-employment to jobs by education and occupation are shown in section 4.

levels. For workers with a college degree the decline in demand for mid-skill jobs instead means that they have fewer opportunities to take a stepping stone job. Therefore, once they find employment they are on average more likely to be employed further up the job ladder, and thus they are less likely to move up. However, as workers compete with each other for jobs it is not only the demand for jobs, but also the supply of educated workers that is linked to mobility and employment. To highlight that transition rates and employment patterns are tightly linked, see figure 3 and figure 4 for changes in job finding rates from unemployment and job-to-job transition rates respectively. The changes in job finding rates from unemployment, unconditional as shown in panel 3a and conditional on education shown in panel 3b closely resemble the changes in the overall employment pattern. While job-to-job transitions share the same relative pattern of changes, the average change is clearly negative. In particular, job-to-job transitions towards non-routine cognitive jobs have seen a decline despite almost no change in job finding rates from unemployment. In the following, I will interpret these changes through the lens of a job search model, that takes into account the heterogeneity across both workers and jobs.

3 Framework

In this section I develop an equilibrium framework of the labor market incorporating (skill) heterogeneity across workers and (technology) differences across jobs. The framework allows for sorting and endogenous mobility of workers. Output from different occupations is aggregated into a final good with a finite elasticity of substitution. As I focus on the stationary equilibrium of the model I drop time as a subscript. Further, I drop individual indices as subscripts in anticipation of the allocation being independent of individual identity, as in Shimer (2005a).

Agents and Technology. Time t is continuous. There is a measure one of risk-neutral workers in the economy. Workers differ in their level of skill indexed by $x = 1, \cdots, X$ which has an exogenous distribution $G(x)$. A worker is either unemployed and searching for a job, or employed and (potentially) searching for another job. The worker chooses search effort s at cost $c(s)$, which is increasing and convex. The search effort cost is allowed to depend on employment status, to capture potential differences in the level of search costs on and off the job. Each unit of search effort translates into a proportional increase in the job finding rate. Workers can also direct their job search. They observe the distribution of vacant jobs and choose to which vacancy to apply for. In contrast to common directed search setups, often based on Moen (1997), I model the application as a discrete choice with idiosyncratic tastes across vacant jobs that follow a Gumbel distribution. This implies, that the elasticity of the application probability with respect to application values is finite. Furthermore, I assume that workers can not coordinate their applications, that is application strategies treat two vacancies with the same characteristics in the same way[5]. This assumption gives rise to matching frictions, as identical vacancies receive zero, one or many applications. Therefore some vacancies remain unfilled, while other vacancies have to turn away applicants.

There is a large measure of potential jobs. Each job chooses its occupation y before entry. There are $y = 1, \ldots, Y$ occupations, which are ordered by their skill intensity y. The productivity of labor depends upon 1) both the workers type x and the jobs occupation y and is captured by the production function $q(x, y)$, and 2) match-specific productivity ϵ, which is redrawn at rate θ from the distribution $F(\epsilon)$. At match formation an initial value of the match-specific productivity is drawn from a distribution $F_0(\epsilon)$, which potentially depends upon the previous employment status of the worker and the type of job. The flow revenue of a job of type y employing a worker of type x is flow output times price $p_y \epsilon q(x, y)$. The price of output p_y of an occupation is determined in equilibrium by market clearing in the competitive occupation output market. A new job opens by posting a vacancy at flow cost $k(y)$. The amount of entry of vacant jobs into the different occupations will be determined in equilibrium and the price of occupation output will adjust accordingly. The output of individual jobs within an occupation are perfect substitutes. Thus, occupation output follows as the sum of output from all individual jobs in an occupation: $Q_y = \sum_x \sum_i \epsilon_i f(x, y) e(x, y, \epsilon)$. The output of each occupation is turned into a single final good Q_F by competitive final goods producers operating a CES technology with elasticity of substitution σ, that is $Q_F = \left[\sum_y \omega_y Q_y^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$. The productivity parameters ω_y govern overall productivity and the relative importance of the different occupations in final goods production. The markets for occupation output Q_y and the final good are competitive. Therefore, the input costs of final goods producers will exhaust revenue. The final good is the numeraire $p_F = 1$.

Labor Market Frictions and Search. Meetings between workers and jobs are stochastic and are modeled by an urn-ball matching function, most closely related to the static setup in Shimer (2005a)[6]. A model with similar features was also studied in Shi (2002). A worker applies for jobs sequentially, but many applications potentially arrive simultaneously at a job. Jobs hire their preferred candidate, as they can only hire one worker. In contrast to a setup where the arrival of applications is fully sequential, the job finding rates of workers do not only depend upon the overall tightness of the labor market, but also on their ranking among the set of applicants. In order to incorporate on-the-job search, which alters the outside option of a worker at time of hiring, I extend the worker-type space. A worker is described by a tuple (x, S_o) where S_o denotes the value of his outside option over unemployment. For simplicity, I assume that match-specific heterogeneity is drawn after hiring decisions have been made. Thus, the set of workers ranked above worker (x, S_o), denoted by $B_y(x, S_o)$, does not depend on ϵ. Next, I derive the job finding rate for a worker of type x, S_o who sends an application to a job of type

[5]See Peters (1991) for the original treatment and Shimer (2005a) for a discussion of this assumption in a closely related setting.

[6]See Peters (1991) and Burdett, Shi, and Wright (2001) for microfoundations of the urnball matching function. See Shimer (2005a) and Shi (2002) for closely related settings.

y. I define a queue of workers $\lambda_y(x, S_o)$ as the effective number of searchers of type (x, S_o) applying for type y vacancies over the number of vacancies v_y. Then, we can also define the queue of better ranked workers

$$\Lambda_y(x, S_o) = \sum_{(x', S'_o) \in B_y(x, S_o)} \lambda_y(x', S'_o).$$

The flow job finding rate at jobs of type y for worker of type (x, S_o) is then

$$\nu_y(x, S_o) = e^{-\Lambda_y(x, S_o)} \frac{1 - e^{-\lambda_y(x, S_o)}}{\lambda_y(x, S_o)} E_\epsilon\{S(x, \epsilon, y) > S_o\},$$

which follows from the urn-ball matching function assumption. The first term represents the probability that no better ranked worker arrives, and the second term represents the probability that the worker will be chosen by the employer among applicants with the same characteristics. Here it is worth to note that for a worker of a given type, the job finding rate does not depend directly on the number of applications by workers who are ranked lower then themselves. The filling rate for a job of type y by a worker of type (x, S_o) follows as $\nu_y(x, S_o)\lambda_y(x, S_o)$, as the urn-ball matching function has constant returns to scale. The overall job filling rate simply follows as the sum of filling rates over worker types. The actual, realized job finding rate for a worker not only depends upon ν_y and the choice where to apply to, but also her total search effort $s(x, S_o)$. Search effort translates one-to-one into job finding rates. Job separations happen at an exogenous rate δ and potentially when a draw of match-specific productivity is so low that the match can not be sustained. These decisions will be described next.

Individual Decision Problems and Bellman Equations. I denote the value of unemployment by $U(x)$, the value of a vacant job of type y by $V(y)$, the value of a filled job by $J(x, S_o, \epsilon, y)$ and the value of employment for a worker in job y by $W(x, S_o, \epsilon, y)$. Furthermore, I will denote deviations of values relative to outside options by hats, that is $\widehat{W}(x, S_o, \epsilon, y) = W(x, S_o, \epsilon, y) - S_o$. The surplus value of a match is defined as $S(x, S_o, \epsilon, y) = W(x, S_o, \epsilon, y) + J(x, S_o, \epsilon, y) - U(x) - V(y)$. The surplus value relative to the outside option is then $\widehat{S}(x, S_o, \epsilon, y) = S(x, S_o, \epsilon, y) - S_o$.

Workers choose how much search effort to exert and which type of job to apply to. Vacant jobs choose which types of contracts to post. Contracts are assumed to be complete and enforceable, thus jobs and workers can commit to fulfilling the conditions of the contract. To describe a workers search decisions define the value of one unit of search effort spent on applications at job type y

$$\Omega_y(x, S_o) = \underbrace{\nu_y(x, S_o) E_\epsilon \widehat{W}(x, S_o, \epsilon, y)}_{\overline{\Omega_y(x, S_o)}} + \mu_y, \tag{1}$$

where μ_y is the idiosyncratic taste for job types, assumed to be i.i.d. across individuals and time. Further μ_y follows a Gumbel distribution with the scale parameter normalized to one and the location parameter set such that the expected value of an application is zero, if the common term $\overline{\Omega_y(x, S_o)}$ is zero. Workers freely choose to which type of job to apply to and draw uniformly one of the vacancies, which offer the same value. The continuous distribution of idiosyncratic tastes implies that generically each applicant will have one preferred type of vacancy and the measure of applicants who are indifferent is zero.

Given the assumption of Gumbel distributed tastes over job types, one obtains the following logit formulation for the share of applications to each type of job

$$P(\text{apply to y}|x, S_o) = \frac{e^{v_y \overline{\Omega_y(x,S_o)}}}{\sum_{y'=1}^{Y} e^{v_{y'} \overline{\Omega_{y'}(x,S_o)}}}, \tag{2}$$

which also accounts for the number of vacancies v_y for each job type. While equation (2) describes the distribution of application choices, it is still necessary to describe the amount of search effort exerted by workers. For simplicity, I assume that workers first choose effort and then realize their idiosyncratic taste for a job type. This implies that the search effort is chosen based on the expected value of search, which follows as

$$\Omega(x, S_o) = \int \cdots \int \max\{\overline{\Omega_y(x, S_o)}\}_{y=1}^{Y} \, dF(\mu_1) \ldots dF(\mu_Y) \tag{3}$$

$$= \log \left(\sum_{y=1}^{Y} e^{\overline{\Omega_y(x,S_o)}} \frac{v_y}{\sum_y v_y} \right), \tag{4}$$

because the distribution of μ_y is Gumbel and i.i.d. across job types.

The workers search effort solves

$$\max_s s\Omega(x, S_o) - c(s),$$

which has an interior solution $s > 0$ as $c(s)$ is assumed to be increasing, monotone and convex. This implies that job search effort will be declining in the rung of the job ladder a worker is currently on, consistent with evidence by Faberman, Mueller, Şahin, and Topa (2022).

The vacant jobs contract posting decision maximizes expected discounted profits. The expected discounted revenue of filling the job is the flow rate at which the job is filled times the total surplus value left after compensating the worker for his outside option. However, a job does not enjoy the remaining value $\widehat{S}$ by itself, but posts contract values $\widehat{W}$ under commitment that promise the worker a specific amount of the remaining value conditional on his characteristics. Following Shimer (2005a) I will formulate the decision problem of the vacant job as one of attracting queues of workers instead of maximizing over contract values directly. The contract values are defined implicitly. Workers first choose which job type to apply to, they then need to decide which vacancy to apply to. Therefore, workers will only apply to the vacancy with the highest expected application value, which is denoted by $\overline{\Omega_y(x, S_o)}$.

$$\max_{\{\lambda_y(x,S_o)\}} \sum_{x,o} \lambda_y(x, S_o)\nu_y(x, S_o)E_\epsilon[\widehat{S}(x, S_o, \epsilon, y) - \widehat{W}(x, S_o, \epsilon, y)] \tag{5}$$

using the definition of expected search value of a job type (1) we can rewrite as

$$\max_{\{\lambda_y(x,S_o)\}} \sum_{x,o} \lambda_y(x, S_o)\nu_y(x, S_o)E_\epsilon[\widehat{S}(x, S_o, \epsilon, y)] - \sum_{x,o} \lambda_y(x, S_o)\overline{\Omega_y(x, S_o)}, \tag{6}$$

where the firm takes the expected search value $\overline{\Omega_y(x, S_o)}$ as given and $\lambda_y(x, S_o) \geq 0$.

The corresponding set of first order conditions is

$$\begin{aligned} \overline{\Omega_y(x, S_o)} \geq\ & e^{-\lambda_y(x,S_o)}e^{-\Lambda_y(x,S_o)}E_\epsilon\left[\widehat{S}(x, S_o, \epsilon, y)\right] \\ & - \sum_{x',o'} e^{-\Lambda_y(x',S_{o'})}(1 - e^{-\lambda_y(x',S_{o'})})\mathbf{1}\{E_{\epsilon'}[\widehat{S}'] < E_\epsilon[\widehat{S}]\}E_\epsilon[\widehat{S}(x', S_{o'}, \epsilon', y)] \end{aligned} \tag{7}$$

$$\lambda_y(x, S_o) \geq 0, \tag{8}$$

where the two conditions hold with complementary slackness.

If no application is attracted then $\lambda_y(x, S_o) = 0$ and equation 7 does not need to hold with equality, but this indeterminacy is without any consequence as no worker of that type applies to this type of job. Replacing $\overline{\Omega_y(x, S_o)}$ with its definition in (1) one obtains a definition of expected contract values as a function of queue lengths. I assume that contracts are complete and enforceable, such that they can not only specify the value promised to the worker, but also on-the-job search and continuation decisions in case of match specific productivity shocks. Therefore, contracts will be specified to maximize the total value of the match. See Garibaldi, Moen, and Sommervoll (2016) for a setup with a similar assumption. Contracts will maximize surplus, so we do not need to specify the value of the match separately for the worker and firm. It is sufficient to describe the joint surplus to describe allocations, as the surplus value does not depend on its split between worker and firm. Continuation decisions of matches can be summarized as follows: a match is discontinued if 1) surplus turns negative or 2) a worker receives a job offer by a job with larger surplus. The Bellman equations defining the values thus follow as

$$rU(x) = \max_s b(x) + s\Omega(x, 0) - c(s, x) \tag{9}$$

$$rV(y) = -k(y) + \sum_{x,o} \mu_y(x, S_o, \epsilon)E_\epsilon\widehat{S}(x, S_o, \epsilon, y) - \sum_{x,S_o} \lambda_y(x, S_o, \epsilon)\overline{\Omega_y(x, S_o)} \tag{10}$$

$$(r + \delta)S(x, \epsilon, y) = \max\{0, \max_s p_y\epsilon q(x, y) + s\Omega(x, S_o) - c(s, x, y) - b(x) - s(x, 0)\Omega(x, 0) - c(s(x, 0), x) \tag{11}$$

$$+ \theta_y \sum_{\epsilon'} F_y(\epsilon')) \max\{S(x, \epsilon', y) - S(x, \epsilon, y), 0\}\}. \tag{12}$$

Here I already set the derivative of the values with respect to time to zero, as I focus solely on stationary equilibria. See appendix A.1 for omitted derivations.

Distribution of Workers and Jobs. Denote the unemployment rate of workers with skill x by $u(x)$ and employment in job of type y and match-specific productivity ϵ by $e(x, y, \epsilon)$. The hiring rate of a worker of type (x, S_o) at a job of type y, while drawing ϵ, is $s_y(x, S_o)\nu_y(x, S_o)f(\epsilon)$ and the rate of separation to unemployment of type x workers at type y jobs is denoted as $\delta_y(x) = \delta + \lambda_y F_y(\underline{\epsilon}(x, y))$. Then the distribution of workers across unemployment $u(x)$ and jobs $e(x, y, \epsilon)$ evolves according to

$$\dot{u}(x) = -\,u(x)\sum_y \int s_y(x,0)\nu_y(x,0)E_\epsilon[\{S(x,\epsilon,y)>0\}] + \sum_{y,\epsilon}\delta_y(x)e(x,\epsilon,y) \tag{13}$$

$$\begin{aligned}\dot{e}(x,\epsilon,y) = &-\,e(x,\epsilon,y)\left(\sum_{\epsilon',y',S'>S} s_{y'}(x,S)\nu_y(x,S)+\delta_y(x)\right) \\ &+\lambda_y f_y(\epsilon)\sum_{\epsilon'} e(x,\epsilon',y) \\ &+\sum_{\epsilon',y',S'<S} s_y(x,S')\nu_y(x,S')e(x,\epsilon',y').\end{aligned} \tag{14}$$

$$G(x) = u(x) + \sum_{y,\epsilon} e(x,\epsilon,y) \tag{15}$$

A stationary distribution satisfies the above law of motion with $\dot{u}(x) = 0$ and $\dot{e}(x, \epsilon, y) = 0$.

Goods Market The final good is a CES aggregate of the occupation output $y_F = \left[\sum_y \omega_y Q_y^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$. The intermediate inputs Q_y, the occupation level output, is bought at price p_y, which is taken as given by competitive final goods producers. Intermediate input demand follows

$$Q_y = \omega_y \left(\frac{p_y}{p_F}\right)^{-\sigma} Q_F. \tag{16}$$

The price index of the final good is

$$p_F = \left[\sum \omega_y p_y^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{17}$$

Costinot and Vogel (2010) provide a model with a similar production structure, but analyze solely the assignment between workers and jobs without labor market frictions.

Stationary Equilibrium

Definition 1. *A pair* $\{U(y), V(y), S(x, \epsilon, y), u(x), e(x, \epsilon, y), \widehat{W}(x, S_o), \lambda_y(x, S_o)\}$ $\forall x, o, \epsilon$ *is a stationary equilibrium, if:*

1. *the workers application probabilities by job type fulfill condition* (2)*;*

2. *the Bellman equations* (9), (10) *and* (12) *hold;*

3. *there is free entry of jobs, that is* $V(y) = 0 \, \forall y = 1, \ldots, Y$*;*

4. *the distribution of workers and jobs is constant over time, that is* (13) *holds with* $\dot{e} = 0$ *and* $\dot{u} = 0$*;*

5. p_y*, the price of occupation output, is such that* (16) *holds with* $Q_y = \sum_x \int \epsilon f(x, y) e(x, \epsilon, y) \, d\epsilon$*;*

6. *the market for applications clears, that is* $s(x, S_o)h(x, S_o) = \sum_y \lambda_y(x, S_o) v_y \, \forall x, o$*;*

7. (7) *and* $\lambda_y(x, S_o) \geq 0$ *hold with complementary slackness.*

Computation of equilibria is implemented using the solver for mixed complementarity problems by Ferris and Munson (1999).

Special Case: Purely Directed Search The model as presented above features idiosyncratic tastes affecting application decisions, which implies that there is a finite elasticity of workers application decisions with respect to the expected value of applying to a vacancy of a particular job type. To make comparison with the previous literature easier, in particular with Shimer (2005a), Costinot and Vogel (2010) and Shi (2002), I present here the limit case when idiosyncratic tastes are absent. As workers freely choose to which type of job to apply to, they will only apply to a job of type y if the application has at least as much value as their second best option.

$$\Omega_y(x, S_o) \geq \max_{y'} \Omega_{y'}(x, S_o) \; \perp \; \lambda_y(x, S_o) \geq 0 \tag{18}$$

This implies that in equilibrium all jobs that attract a given worker type offer the same expected value of an application. Thus, $\overline{\Omega_y(x, S_o)} = \overline{\Omega(x, S_o)} = \Omega(x, S_o)$. If two job types offer the same expected value of an application for a worker, the equilibrium features mixed strategies for applications. This version of the model is useful as a benchmark case to highlight the determinants of sorting, but is too restrictive to fit the data.

In the following, I present example allocations to clarify basic properties of the model and for this I focus on the version where idiosyncratic tastes for job types are absent.

3.1 Examples: Sorting

In the model are several forces that drive sorting and in this section I give examples to clarify those mechanisms. First, I focus on the role of job output $f(x, y)$. Second, I will discuss the role of entry costs. Here sorting is defined as first order stochastic dominance.

Definition 2. *An allocation exhibits positive assortative matching (PAM), if the distribution over jobs y for workers of type x_2 first order stochastically dominates that of workers with type x_1 when $x_1 < x_2$.*

1. *The conditional distribution of employment across jobs for a worker of type x is $\pi(y|x) = \frac{\sum_{j=1}^{y} \int e(x,\epsilon,j)d\epsilon}{\sum_{j=1}^{Y} \int e(x,\epsilon,j)d\epsilon}$*
2. *An allocation exhibits PAM, if $\pi(y|x_i) \leq \pi(y|x_{i'})$ $\forall i, i' > i \in 1, \cdots, X$ with the inequality strict for at least one $y \in \{1, \ldots, Y\}$ and $x_i < x_{i'}$.*

Negative assortative matching (NAM) is defined analogously. Note that I define sorting globally across all pairs of workers. To clarify under which conditions sorting occurs it is useful to consider when no sorting occurs.

Proposition 1. *Assume match productivity $q(x, y)$ is log-modular, entry costs are independent of job type $k(y) = k_0$ and the distribution of ϵ is independent of job type. Then no sorting according to definition 2 occurs in a stationary equilibrium. Proof see appendix.*

The no sorting condition is the same as in the frictionless case when $k_0 \to 0$. In the frictionless limit there are no wage differences across jobs, but even in the case with frictions a similar condition holds in terms of surplus. Thus, for models similar Shimer (2005a) to explain evidence if differences in wages across jobs for similar workers in matched employer-employee data, one needs to assume that firms have different entry costs in order to sustain surplus differences across jobs in equilibrium.

For the following examples, consider a simplified version of the model above where the only form of heterogeneity is in worker skill x and job type y. There are two types of workers $x_L < x_H$ and two types of jobs $y_L < y_H$, and there is no match specific productivity variation. The home production value is $b(x) = \bar{b}$. In that case, surplus follows

$$(r + \delta)S(x, y) = p_y q(x, y) - \bar{b} - s(x, 0)\Omega(x, 0) + c(s(x, 0)) - s(x, S)\Omega(x, S).$$

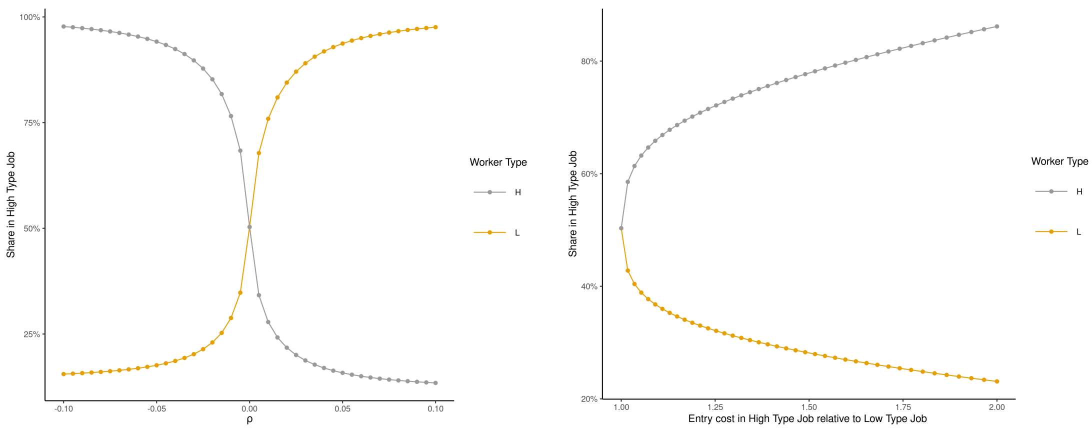
The production technology is

$$q(x, y) = [x^\rho + y^\rho]^{\frac{1}{\rho}},$$

where ρ governs whether skill x and job type y are complementary. In general, the properties of surplus $S(x, y)$ determine sorting, which is true also in random search settings (Postel-Vinay, Lindenlaub et al., 2017). Shimer (2005a) discusses some examples under which sorting arises in a setup that also features directed search and screening by employers, while taking surplus differences as given. In contrast, in the present analysis there is free entry and therefore higher surplus in some types of job are only sustainable to the extent that they reflect lower filling rates μ_y or higher posting cost k. In equilibrium additional entry will lead to a decrease in p_y up until the free entry condition is satisfied. In the following I give examples in which sorting occurs as a function of the underlying production technology and entry costs across jobs, instead of focusing on the realized surplus differences.

Comparative Advantage in Production. Assume $k(y) = k_0 \, \forall y$. Then, the conditions for sorting are the same, as in the frictionless limit $k_0 \to 0$. Costinot and Vogel (2010) show that in the frictionless assignment model sorting arises when $f(x, y)$ is log-supermodular, that is high skill workers have a comparative advantage in high skill intensive occupations. In the current example, the production function is log-supermodular if $\rho < 0$. In the two-type example we can summarize the distribution of workers across jobs, as the share of workers in high type jobs $\pi_H(x) = \frac{e(x,y_H)}{e(x,y_L)+e(x,y_H)}$. Figure 5a plots $\pi_H(x)$ for low and high skilled workers for various values of ρ in a numerical example. The condition for PAM is satisfied if $\pi_H(x_H) > \pi_H(x_L)$. PAM occurs in equilibrium when $f(x, y)$ is log-supermodular. In this example with a CES production function, log-supermodularity holds when $\rho < 0$. When $\rho = 0$, there is no sorting and when $\rho > 0$ ($f(x, y)$ is log-submodular) the allocation exhibits NAM.

Figure 5: Sorting with comparative advantage and heterogenous entry costs.



(a) Employment Share in y_H Jobs and ρ. Log-supermodular (-submodular) production function $\rho < 0$ ($\rho > 0$) implies PAM (NAM). Entry Cost $k(y) = k_0$.

(b) Employment share in y_H jobs with heterogenous entry cost $k(y) = k_0 y^{k_1}$. Increasing entry cost in job type, $k_1 > 0$, implies PAM in absence of comparative advantage $\rho = 0$.

The reason that the condition for sorting is not stronger with frictions in the labor market relative to the frictionless case is that jobs select workers at the hiring stage. When they receive multiple applications, they hire the worker delivering the highest value to the firm, which coincides with the worker who provides the highest surplus. Therefore, when deciding which worker to hire the firm ranks according to the same criterion as in the frictionless case and sorting arises under the same conditions. However, there is mismatch. Some firms receive only applications by L type workers, while others only receive applications by H type workers. Therefore, sorting is not perfect as it would be in the frictionless case. Mismatch is sustained in equilibrium despite directed search, because firms post contracts conditional on worker heterogeneity rendering workers indifferent between applying at different jobs.

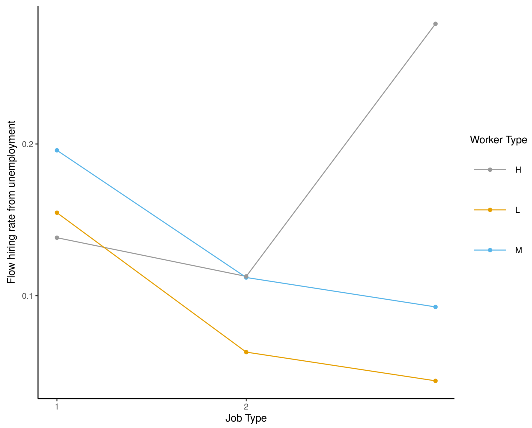
Heterogeneous Entry Cost. Differences in entry costs across occupations y induce sorting, even when the production function is log-modular. The reason is that differences in entry costs are reflected in surplus values due to free entry. However, those differences are larger for more skilled workers even in absence of comparative advantage ($\rho = 0$). Consider the same setup, as in the previous example, but with $k(y) = k_0 y^{k_1}$. Figure 5b plots the share of employment in high skill jobs π_H for low and skill workers. When high skill jobs are more costly to create, $k_1 > 0$, the equilibrium exhibits PAM even with a log-modular production function. When entry cost are increasing in y the productivity advantage of y_H jobs is not fully competed away due to entry. When $k_1 > 0$, the relative price of output of y_H jobs is larger compared to an equilibrium with $k_1 = 0$. As the price of output for high type jobs does not fall as much, surplus can be supermodular without $f(x, y)$ being log-supermodular.

3.2 Examples: Allocation

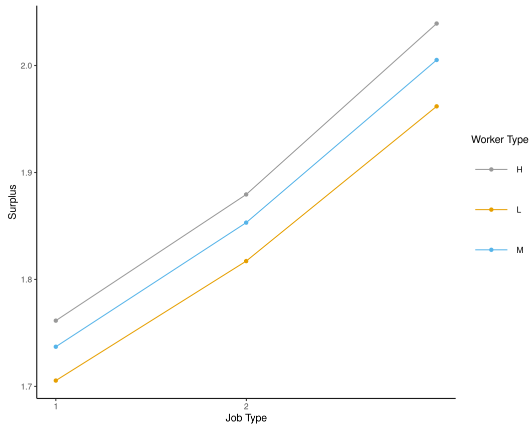
In this section I present an example allocation to illustrate how job finding rates are affected by competition between workers, not just surplus value of jobs. In this example I maintain the setup of the previous subsection and focus on the case with three types of workers and three types of jobs. The allocation features positive assortative matching as defined above. See appendix B for the full list of parameters.

Panel 6a shows the flow job finding rate of L,M and H type workers at L,M, and H type jobs from unemployment. The parameters are chosen so there is positive assortative matching. All types of workers match with all types of jobs. Thus, compared to a frictionless allocation there is mismatch, even with directed search. Further, the sorting patterns are apparent directly in the transition rates from unemployment to employment. That is H type workers match mostly with H type jobs and L type workers mostly with L type jobs. However, surplus is increasing in job type for all types of workers, as shown in Panel 6b. Thus, job finding rates do not reveal directly surplus. In equilibrium, low skilled workers apply at a low rate to high type jobs, because the contract offered to them does not compensate them enough for the increased competition by high skilled worker, which lowers their job-finding rate. Panel 6c shows the probability that the no worker with a a type that is preferred by the employer also applies for the same vacancy. Thus a decreasing probability indicates more competition by higher ranked workers. Thus, a worker does not only care about the ex-post payoff of a match, but also the difficulty of obtaining the job. This is a standard feature of search models of the labor market, but the dependence of the job finding rate on the *composition* of the applicant pool is often

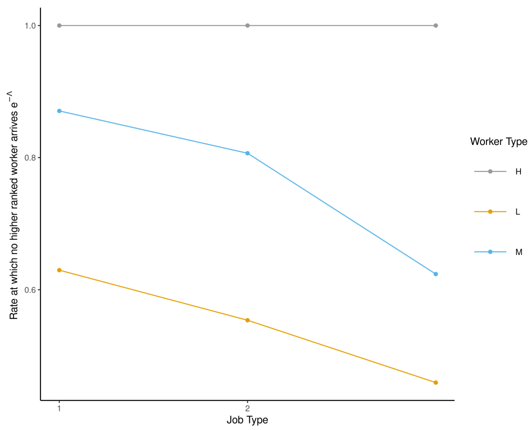
Figure 6: Job Finding Rates, Competition and Directed Search



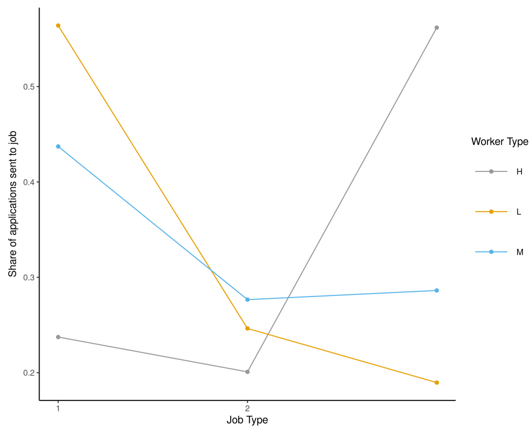
(a) Flow job finding rate of L,H type workers at L,H type jobs: $s_y(x,0)\nu_y(x,0)$



(b) Surplus: $S(x,y)$



(c) Flow rate of applications at which the worker is the best applicant: $e^{-\Lambda_y(x,0)}$



(d) Share of applications to job type y by worker of type x

ignored by assuming a fully sequential arrival of applicants at vacancies. To complete the description of the matching process Panel 6d shows the flow rate of applications for each type of job. L type workers apply predominantly for L type jobs and H type workers apply predominantly H type jobs. These probabilities are equilibrium objects and depend on the interaction of application decisions of job seekers and labor demand for each type of worker at each type of job. Here I focused on workers who are looking for jobs from unemployment, but the exact same mechanism applies for all searchers independent of employment status.

4 Estimation

The main data sources used in the estimation were described in section 2. For the estimation I use again the same occupation and education groups. That is there are four occupation groups: non-routine manual (NRM), routine manual (RM), routine cognitive (RC) and non-routine cognitive (NRC). Workers are classified according to their education level into three groups: at most a high school degree (HS), some college (SC) and a four year degree or more college (C).

4.1 Setup

The goal of the estimation is to identify the structural parameters governing production and matching in the economy. The model parameters are estimated by Indirect Inference following Gourieroux, Monfort, and Renault (1993). I pick a set of moments m to identify the model parameters β. The estimation procedure minimizes the weighted square distance between model $m(\beta)$ and data moments $\bar{m}$ by choosing parameters[7].

$$\min_{\beta} \quad \sum_{i} \omega_i (\bar{m}_i - m_i(\beta))^2 \tag{19}$$

where ω_i is a weight on the i-th moment[8]. The estimation is done separately for the period 1996-1998 and 2016-2018, while treating each allocation as stationary. Labor market flow rates are large and therefore the half-life of distributions is short. Thus, treating allocations as approximately stationary is not likely to lead to large errors. In practice the model parameters are estimated following the approach in Chernozhukov and Hong (2003), because local optimization methods prove to be too costly and not sufficiently robust. The simulation of model parameters by Markov Chain Monte Carlo (MCMC) method is done using the robust adaptive metropolis algorithm proposed in Vihola (2012). See appendix C for the calculation of standard errors.

Moments and Identification To estimate the model parameters I mainly use moments on labor market flows. While all moments together inform the parameters, I give a short argument how moments inform specific parameters.

1. In order to inform the productivity parameters $q(x, y)$, ω_y, the home-production values b_x and the level of search costs from unemployment c_0 I use the job finding rates from unemployment, the wages by worker type and lastly a normalization of the average home-production value relative to wages. First note, that out of the production function parameters, I normalize the level of

[7]The model equilibrium is solved for by using the PATH solver (Ferris and Munson, 1999). Furthermore, the model implied stationary distribution is used for calculating model moments

[8]The estimation also includes a normalization that is used in the same way as the data moments in the estimation. Their weight is set to 1e-4. Standard errors are calculated conditional on this constraint

$q(HS, y) = 1$ by job type, because q and ω_y enter as a product into final goods production. This leaves us with 12 production parameters. Further, there are 3 home-production values b_x and one parameter determining the search costs level c_0. Thus in total we have 16 parameters. To identify those, I use the job finding rates from unemployment (12 moments), the average wage by worker type (3 moments) and normalize the average of the home production value relative to wages, which gives the required 16th restriction. Following the arguments in Lise and Robin (2017), the job finding rates can identify the surplus value of jobs given search costs. However, in order to also set the level of productivity and the level of search costs I additionally target the wages by worker type and the normalization of the average home production value.

2. I parameterize the distribution of match-specific productivity shocks as a two-point distribution centered around one, the difference between high and low productivity is $\Delta\epsilon$. For innovations to match-specific productivity the distribution is assumed to be uniform. However, I allow the share of high productivity values at match creation to depend upon the previous employment status of the worker. First, the share of high productivity draws depends upon whether the worker was unemployed previously, which affects the amount of job-to-job transitions in low tenured jobs. Second, for job to job transitions the share of high productivity draws depends upon whether the worker was employed in the same type of job previously. This allows to capture the high tendency of within occupation group transitions. Further, innovations to match-specific productivity follow a Poisson process with arrival rate θ. That is the parameters $\Delta\epsilon$, θ and the share of high innovations by origin of the incoming worker need to be set. These are in total seven parameters. To identify those parameters, I use information on job-to-job transition rates. The share of job-to-job transitions within occupations is used to inform the share of high match values in the offer distribution for within occupation transitions (4 moments to inform 4 parameters). Further, I use the share of job-to-job transitions down the job ladder to inform $\Delta\epsilon$, as larger variation in match-specific values lowers the relative importance of value differences across job types. However, in practice this leaves us with little information above and beyond the remaining moments, thus I add the residual standard deviation of wages within occupation-education group as an additional moment to inform the dispersion in match specific productivity (2 moments to inform 1 parameter). The arrival rate of innovations to match specific productivity θ and the share of high match productivity draws from unemployment are jointly informed by the the pattern of job-to-job transitions by tenure. I use job-to-job transitions in the first two years and third to fourth year relative to transitions in years five to ten as moments. For high-tenured workers innovations to match specific productivity are the main determinant of job-to-job transitions. For low tenured workers the quality of the initial match is an important determinant, thus it is strongly affected by the share of workers that are receive a high productivity draw. The arrival rate of productivity innovations θ affects how long the initial match quality matters on average,

thus affecting how fast job-to-job transition rates change with tenure (2 moments to inform 2 parameters).

3. Entry costs $k(y)$ are allowed to depend upon job types, in order to allow for persistent differences in job values despite free entry. I use the overall ratio of hires to vacancies and job-to-job transition rates by destination occupation to inform entry costs. Vacancies are not measured by occupation in the data, therefore I only use a measure of total vacancies relative to the labor force (Barnichon, 2010), which informs the average entry cost. The relative job-to-job transition rates by job type can inform the differences in value across jobs, as job-to-job transitions are directed to higher value jobs, a higher inflow rate of job-to-job transitions reveals a higher value, which under free entry is only present if the entry costs for that occupation are larger. (4 moments to inform 4 parameters).

4. To allow the model to fit job-to-job transition rates by worker and job type even if the model misses some heterogeneity in mobility determinants, I allow the search cost parameter ϕ to be different when employed and to depend upon the worker type and job type: $\phi = \phi_1(x)\phi_2(y)$. The search costs are directly related to the effort exerted in job search and thus directly affect transitions rates (6 moments to inform 6 parameters).

5. The elasticity of substitution between job types σ is calibrated separately from the estimation. I use a value of $\sigma = 0.9$ based on Goos et al. (2014).

6. The share of workers by education level $G(x)$ as estimated from the data is directly imposed. See table 9a in appendix section D for the values.

7. Transition rates from employment to unemployment by job and worker type $\delta(x, y)$ are also directly imposed in the model. See table 9b in appendix section D for the values.

Table 2: External Targets

Target	Value	Description
σ	0.9	Elasticity of substitution between job types (Goos et al., 2014)
$\bar{b}$	50% of mean monthly earnings	Normalization of average home production value (Lise and Robin, 2017, Shimer, 2005b)

Tables 3 and 4 summarize the moments used for estimating the model parameters and present them alongside the model moments at the estimated parameters for both 1997 and 2017. Panel 3a shows unemployment to employment transition rates by education and destination occupation. First note, that it became substantially less likely for workers to find routine type jobs. This decline has been particularly strong for high-school graduates in routine manual jobs and in routine cognitive jobs for workers with some college education. On the other hand, for workers of all education level it became more likely to start working in a non-routine manual job. As already highlighted in section2 the polarization of employment is not directly apparent conditional on education levels, but is consistent with the aggregates due to the substantial upward shift in educational attainment.

Panel 3b shows average weekly earnings in 2015 US$[9]. Average earnings rise slightly for high school graduates, fall slightly for workers with some college education and rise for college educated workers.

Now I turn to the moments related to the mobility of workers between jobs and the overall hiring rate in table 4. In panel 4a job-to-job transitions by education level are shown. For all education levels transitions fall substantially by approximately 20%. Turning to the direction of job-to-job transitions in panel 4b I present the relative amount of job-to-job hires and separations by occupation group, with the reference group being non-routine manual jobs. Relative job-to-job hiring rates have fallen substantially for routine jobs, similar to hiring rates from unemployment. In contrast job-to-job separations have remained similar in relative terms. These moments indicate that changes in the overall employment patterns across jobs are also present in hiring rates, in line with the evidence by Cortes et al. (2020) on the importance of inflow rates for changes in employment patterns. Next in panel 4c I present the overall hiring rate relative to the level of vacancies and additional moments related to job to job transitions. The hiring rate relative to vacancies has fallen over time, which indicates that it takes longer to fill any given job over time. The share of job-to-job transitions that move to a lower job type is approximately 15% and does not change between 1997 and 2017. The categorization of jobs is based on the previous literature on job polarization, i.e. interpreting the order given as a ranking is ad-hoc, but the low share of transitions moving down the ladder indicates, that the ranking has reasonable content. Another important feature of job-to-job transitions is, that they are highly concentrated to similar jobs as shown by the share of transitions that occur within the same job category. This share is lowest in non-routine manual and routine cognitive jobs, with approximately 60% and highest in routine manual and non-routine cognitive jobs with over 70%. These moments indicate that transitions are highly concentrated within job types. In line with the decline in transition rates to routine jobs, the share of transitions within occupation categories is also falling for routine occupations over time, but slightly rising in non-routine occupations. Lastly, I present the change of job-to-job transitions over the tenure distribution. Job-to-job transition rates are particularly high in the first two years of employment and fall already by the third year to a low and almost stable level. The number of transitions in the first two years of employment has increased over time, relative to transitions in years 5-10 of employment. Overall, these moments indicate that in particular hires by occupation group, both from unemployment and other employers, have changed substantially over time. Additionally, strong sorting by education and occupation group coexists with substantial mobility across occupation groups. Therefore, it is important that the proposed framework incorporates forces that lead to sorting in a frictional labor market.

[9]To deflate the values to 2015 values I use the Consumer Price Index for All Urban Consumers: All Items (CPIAUCSL) provided by the Bureau of Labor Statistics.

Table 3: Targeted Moments (i)

(a) Unemployment-Employment Transition Rate

Education	Occupation	Model 96	Data 96	Model 16	Data 16
High School	non-routine manual	7.16	7.04	8.91	8.8
			(0.34)		(0.42)
	routine manual	15.63	15.72	13.13	13.18
			(0.44)		(0.47)
	routine cognitive	5.34	5.3	4.96	4.91
			(0.28)		(0.37)
	non-routine cognitive	1.71	1.67	1.74	1.79
			(0.14)		(0.17)
Some College	non-routine manual	5.78	5.88	7.77	7.73
			(0.34)		(0.49)
	routine manual	11.5	11.68	9.41	9.63
			(0.56)		(0.49)
	routine cognitive	10.72	10.93	8.22	8.27
			(0.6)		(0.47)
	non-routine cognitive	7.17	7.16	7.06	7.06
			(0.5)		(0.47)
College	non-routine manual	2.83	2.95	3.84	3.81
			(0.33)		(0.33)
	routine manual	4.04	4.1	3.33	3.35
			(0.36)		(0.28)
	routine cognitive	9.5	9.63	7.18	7.06
			(0.74)		(0.49)
	non-routine cognitive	21.08	21.17	22.35	22.28
			(0.91)		(1.18)

(b) Average weekly Earnings by Education

Education	Model 96	Data 96	Model 16	Data 16
High School	673.0	671.0	680.0	682.0
		(3.0)		(3.0)
Some College	808.0	810.0	785.0	787.0
		(4.0)		(3.0)
College	1189.0	1193.0	1252.0	1251.0
		(8.0)		(4.0)

Notes: Data moments based on own calculations using CPS Basic Monthly Files and Tenure Supplements. Standard errors in parentheses.

Table 4: Targeted Moments (ii)

(a) Job-to-Job Transitions by Education

Education	Model 96	Data 96	Model 16	Data 16
High School	2.5	2.5	2.0	2.1
		(0.047)		(0.056)
Some College	2.4	2.5	2.0	2.0
		(0.058)		(0.058)
College	2.3	2.2	1.8	1.8
		(0.064)		(0.054)

(b) Job-to-Job Hires and Separations by Occupation

	Occupation	Model 96	Data 96	Model 16	Data 16
j2j to occupation	routine manual	2.2	2.1	1.4	1.2
			(0.079)		(0.066)
	routine cognitive	1.8	1.8	0.98	1.0
			(0.063)		(0.057)
	non-routine cognitive	2.2	2.2	2.2	2.1
			(0.077)		(0.11)
j2j from occupation	routine manual	0.98	0.91	1.0	0.96
			(0.031)		(0.044)
	routine cognitive	0.83	0.88	0.86	0.88
			(0.025)		(0.043)
	non-routine cognitive	0.75	0.69	0.78	0.71
			(0.021)		(0.031)

(c) Job Filling Rate, Job Ladder and Job-to-Job separations by Tenure

	Occupation	Model 96	Data 96	Model 16	Data 16
$\frac{\text{hire rate}}{\text{vacancies}}$		51.13	51.8	43.42	41.89
			(0.96)		(1.08)
$\frac{j2j_{y'<y}}{j2j}$		15.56	14.92	15.78	15.14
			(0.36)		(0.58)
$\frac{j2j_{y'=y}}{j2j}$	non-routine manual	57.1	58.67	58.03	61.12
			(1.2)		(1.81)
	routine manual	73.94	74.61	69.64	72.2
			(0.62)		(1.21)
	routine cognitive	61.99	62.39	53.01	53.75
			(0.83)		(1.63)
	non-routine cognitive	73.23	73.21	76.07	77.21
			(0.85)		(1.13)
$\frac{j2j_{t\in[0,2)}}{j2j_{t\in[5,10]}}$		8.16	4.5	9.28	6.04
			(1.36)		(1.44)
$\frac{j2j_{t\in[2,5)}}{j2j_{t\in[5,10]}}$		1.03	1.16	1.13	1.17
			(0.38)		(0.38)

Notes: Data moments based on own calculations using CPS Basic Monthly Files and Tenure Supplements. Standard errors in parentheses.

Parameter Estimates

The estimated parameters are summarized in table 5 and 6 for both 1997 and 2017. The occupation productivity estimates are summarized in panel 5a. The parameter ω_y governs an occupations productivity level. Between 1997 and 2017 the estimated productivity has fallen for routine jobs and risen for non-routine jobs. The changes in estimated productivity by occupation reflect changes in job finding rates. Within the model however it is not only productivity that governs labor demand, but also entry costs k_y. In contrast to overall productivity, estimated entry costs shown in panel 5b have risen for routine and non-routine cognitive occupations, but fallen for routine cognitive jobs. However, the standard errors are particular large for entry costs. An occupations productivity level ω_y and entry costs governed by k_y are the main determinants of overall labor demand by occupation, but the allocation of workers by education to jobs depends also strongly on the workers relative productivity. Panel 5c shows the estimated productivity premium of some college and college education over high school graduates by job type. These estimates indicate that estimated productivity premia have remained almost constant over time. This already indicates that changes in sorting patterns, are likely driven by shifts in overall demand and supply and not by changing productivity premia of education. Further, the estimates indicate that education has the smallest productivity premium in routine manual jobs, consistent with the higher importance of manual vs. cognitive skills in these jobs. Wages and transition rates are also impacted by workers outside options, in particular the home production value b_x. In the estimation the home production value is allowed to vary with the education level of workers, while the average is normalized relative to wages. The panel 5d shows the estimated home production values, which show limited changes constant over time.

Table 5: Parameter Estimates (i)

(a) Productivity by job type ω_y

	non-routine manual	routine manual	routine cognitive	non-routine cognitive
1997	0.37	0.93	0.7	1.7
	(0.07)	(0.065)	(0.076)	(0.63)
2017	0.51	0.74	0.59	2.1
	(0.2)	(0.22)	(0.34)	(0.25)

(b) Entry cost by job type k_y

	non-routine manual	routine manual	routine cognitive	non-routine cognitive
1997	7.1	1.3	0.37	1.6
	(4.1)	(0.54)	(0.21)	(0.66)
2017	4.7	1.7	0.56	1.8
	(3.3)	(0.78)	(0.46)	(1.3)

(c) Productivity premium of education

		non-routine manual	routine manual	routine cognitive	non-routine cognitive
Some College	1997	1.1	1.1	1.1	1.1
		(0.021)	(0.012)	(0.042)	(0.1)
	2017	1.1	1.0	1.1	1.1
		(0.00083)	(0.0047)	(0.11)	(0.041)
College	1997	1.3	1.2	1.4	1.4
		(0.069)	(0.037)	(0.075)	(0.23)
	2017	1.3	1.2	1.4	1.4
		(0.039)	(0.019)	(0.16)	(0.051)

(d) Home Production by education

	High School	Some College	College
1997	0.27	0.4	0.78
	(0.027)	(0.0)	(0.02)
2017	0.25	0.37	0.79
	(0.08)	(0.0)	(0.13)

(e) Match Effects: Productivity Dispersion $\Delta\epsilon$ and poisson arrival rate of shocks θ

	θ	$\Delta\epsilon$
1997	0.01	1.4
	(0.00041)	(0.17)
2017	0.0074	1.1
	(0.00026)	(0.12)

Notes: Standard errors in parentheses.

Match specific productivity is not only driven by worker and job types, but also by a match specific productivity term ϵ that is redrawn at rate θ. The match specific productivity distribution is parameterized as a two-point distribution centered around one. The high and low value have a distance of $\Delta\epsilon$. The dispersion has fallen, but implies that productivity in low value matches is less than one third that of high value matches even in 2017. The positive arrival rate of match specific productivity shocks reflects that job-to-job transitions occur even in high tenured matches. The decline in the arrival rate θ suggests that matches have become more stable over time.

Table 6: Parameter Estimates (ii)

(a) Offer Distribution of match specific productivity $F(H)$ and search costs for unemployed workers $\phi_0 s^2$

	F(H) from u	ϕ_0
1997	0.43	15.2
	(0.04)	(0.9)
2017	0.35	19.33
	(0.06)	(1.58)

(b) Search costs on the job: $\phi(x, y)s^2 = \phi_0\phi_x\phi_y s^2$

	High School	Some College	College
1997	0.19	0.76	0.97
	(0.078)	(0.063)	(0.17)
2017	0.37	0.73	0.74
	(0.3)	(0.054)	(0.094)

	routine manual	routine cognitive	non-routine cognitive
1997	0.58	2.9	0.94
	(0.13)	(0.59)	(0.33)
2017	0.45	2.3	0.59
	(0.096)	(1.3)	(0.28)

(c) Offer Distribution of match specific productivity $F_y(H)$ for job-to-job transitions within job type y'

	non-routine manual	routine manual	routine cognitive	non-routine cognitive
1997	0.88	0.87	0.79	0.75
	(0.11)	(0.16)	(0.03)	(0.096)
2017	0.89	0.86	0.76	0.8
	(0.046)	(0.035)	(0.074)	(0.053)

Notes: Standard errors in parentheses.

Table 6 summarizes the remaining estimated parameters regarding the offer distribution of match specific productivity and the search cost parameter ϕ. In panel 6a the share of high productivity matches for jobs found from unemployment is shown alongside the estimated search cost parameter ϕ_0. Between 1997 and 2017 it is estimated to becomes less likely to find a high value match right out of unemployment, but at the same time search costs have risen. The lower likelihood to find high value matches right out of unemployment is informed by the relative rise in job to job transitions at low tenures, which should not necessarily be interpreted as reflecting an on average worse initial distribution of match quality, as the dispersion in match specific productivity $\Delta\epsilon$ has fallen. The estimated changes are consistent with increased screening by firms, that now select from further into the right tail of an underlying distribution of match qualities. In Panel 6b the search cost parameters for on the job search are shown, which are expressed relative to the search cost parameter from unemployment.

These search cost parameters capture variation in job-to-job transitions over time unexplained by the remaining mechanisms in the model. Finally, in panel 6c the share of high value matches in the offer distribution for job to job transitions within an occupation is shown. Allowing for differences in the offer distribution for within occupation job to job transitions and across jobs is supposed to capture features of the labor market, like job specific skills and job search through networks not included in the model, that make it more likely for workers to transition to similar jobs. The parameter estimates show, that it is important to allow for such differences as the estimated share of high value matches is around 80% in all occupations, much higher then the value of 50% which is set as a normalization for job-to-job transitions across occupations. Further, in line with evidence by Faberman et al. (2022) the share of high value matches is lowest from unemployment and substantially higher for job-to-job transitions.

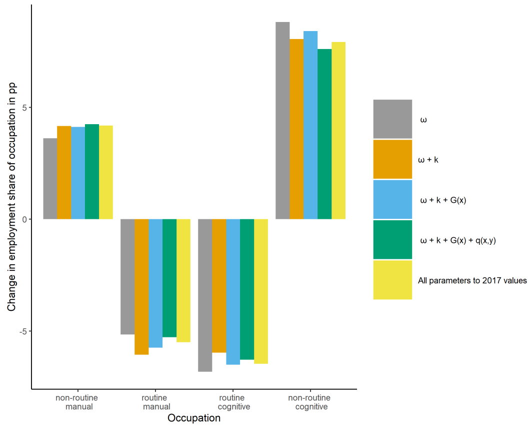
Together with the shift in the education distribution shown in table 9a, the parameter estimates reflect substantial changes in the composition of labor demand, and labor supply. Further, the model captures through the estimated match offer distribution, arrival rate of shocks and search costs further changes in the hiring process. In the next section, I will show which of these changes are driving the observed overall changes in employment, sorting and job-to-job transition rates.

5 Results

In this section I use the estimated model to compare the importance of different channels for the observed changes in the labor market. I focus here on changes in employment by occupation, the sorting of workers by education to the different occupations and for the changes in job to job transition rates. As the baseline allocation I use the model with the estimated parameters for the initial period, 1997. Then I adjust a subset of the parameters to their estimated value for the later period, 2017. First, I focus on the shift in the occupation productivity parameter ω_y and the entry costs k_y, the education distribution of the labor force $G(x)$ and the productivity premium of education by occupation $q(x, y)$. The occupation productivity parameter ω_y and the entry costs k_y capture underlying technological change leading to changes in labor demand by occupation. While $q(x, y)$ allows technological change to be biased towards particular education-occupation pairs. Second, for the decline in job-to-job transition rates I also take into account the change in the match specific productivity distribution, and search costs.

Polarization of employment. Figure 7 shows the change in the employment share of each occupation in percentage points. Each bar represents a separate counterfactual. The first bar shows the result for the counterfactual that only changes the occupation productivity parameter ω_y to their estimated value for 2017 and leaves all other parameters at their 1997 value. The last bar shows the results when all parameters are changed to their 2017 value. These results indicate that solely the change in the occupation productivity parameter can explain most of the shift in employment across the occupation groups. Further, they are in line with a strong focus of the literature on explaining shifts in employment by labor demand changes. However, in section 2 I have shown that the changes in employment in the aggregate are quite different to those by education group. Thus, I turn now to the sorting of workers by education level to jobs.

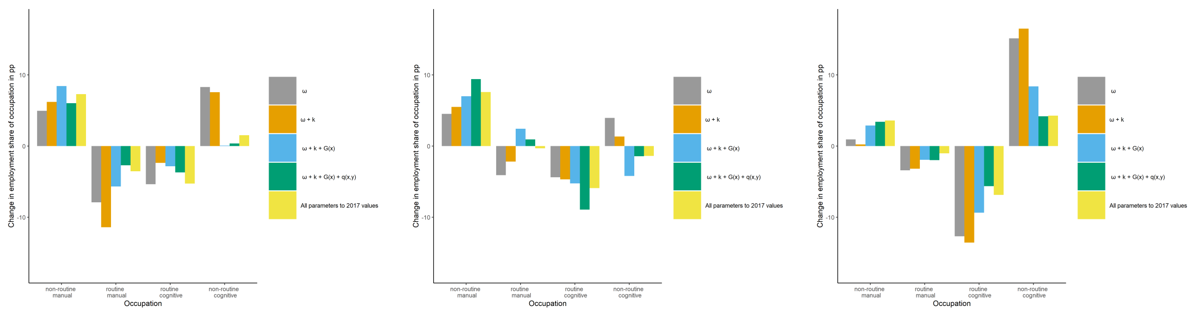
Figure 7: Change in Employment by Occupation



Sorting and polarization of employment. The aggregate changes in employment by occupation group hide substantial heterogeneity in the employment shift by education group. Further, the aggregate changes also hide the impact of labor supply shifts. In figure 8 the dis-aggregated results by education group are shown, each panel refers to one education group. First, the occupation productivity shifter ω_y can not explain by itself the differences in the employment shifts by education group. Just the shift in ω_y would substantially over-predict the rise in employment in non-routine cognitive occupations and under-predict the rise in employment in non-routine manual occupations. Further, for routine manual occupations the shift in ω_y implies a too large fall in routine manual employment. But for routine cognitive jobs the implied fall is over-predicted for college educated workers, while under-predicted for those with some college education. Adding the shifts in entry costs k_y does not change the overall qualitative pattern and has also quantitatively limited impact. However, accounting for the shift in the education distribution $G(x)$ brings the implied employment distribution much closer to the overall change. The shift in labor supply is essential to explain that there has been almost no rise in employment in non-routine cognitive occupations conditional on education level. Further, the shift matters for the allocation of workers to the remaining jobs as well. In particular, increasing education levels push workers with (some) college education also towards non-routine manual jobs. Overall, just the shift in labor demand driven by ω_y and k_y, and the increase in education levels can explain most of the employment shift across occupations by education level. The remaining changes can be explained mostly by the productivity premium of education by job type $q(x, y)$. These results suggest, that it is important to take into account both labor supply and labor demand shifts when studying recent trends in employment. Further, simple descriptive decomposition analysis would not uncover the effects, as the interaction in supply and demand has equilibrium effects on the assignment of workers to jobs. Consistent with Cortes et al. (2017) the shifts in employment can almost fully be explained by changes in inflow rates.

Figure 8: Change in Employment by Occupation and Education

(a) High School Graduates (b) Some College (c) College



Decline in job-to-job transition rate. The previous results focus on employment shifts, however one goal was to quantify the role of shifts in labor demand and supply for the mobility across jobs. To do so, I first present the results on how the shifts in labor demand and supply affect the job-to-job transition rate. Second, I present the effect of a shift in the search costs, changes in the arrival rate and distribution of match effects. These factors are important to explain the overall changes in the job-to-job transition rate. Table 7 shows the counterfactual changes in the average job-to-job transition

Table 7: Decline in job-to-job transition rate

(a) Labor demand and supply counterfactual

	Model 96	ω_y	$\omega_y + k_y$	$\omega_y + k_y + G(x)$	$\omega_y + k_y + G(x) + q(x, y)$	Model 16
j2j	0.0242	0.0239	0.0228	0.0235	0.0235	0.0194

(b) Search costs and match effects counterfactual

	Model 96	$\Delta\epsilon$	$\Delta\epsilon + \theta$	$\Delta\epsilon + \theta + \phi$	$\Delta\epsilon + \theta + \phi + \text{Offer}F(\epsilon)$	Model 16
j2j	0.0242	0.0224	0.0193	0.0196	0.0195	0.0194

rate in the model economy in response to labor demand and labor supply factors, in panel 7a, and in response to shifts in the mobility cost parameters and match effects distribution and arrival rate of shocks in panel 7b. The shifts in labor demand driven by the occupation productivity ω_y and entry costs k_y have a negative effect on the job-to-job transition rate, explaining about 30% of the decline between 1997 and 2017. In contrast, the increasing education levels, represented by the shift in the type distribution $G(x)$, actually implies a rise in the job-to-job transition rate. Thus, taken together the shifts in labor supply and demand have only a small effect on aggregate mobility rates of workers. Table 7b shows that average changes in the mobility rates can be mostly accounted for by the decline in the dispersion of match specific productivity $\Delta\epsilon$ and the decline in the arrival rate of innovations to match specific productivity θ. The decline in dispersion in match specific productivity lowers the dispersion in value of matches that is not directly dependent on worker, or job type. As such, with lower differences in match values the incentive to search for new jobs becomes lower and as a result job-to-job transition rates decline. The arrival rate of innovations to match specific productivity alter in particular the likelihood that workers at higher tenures receive a negative shock to match value, and thus move to a new job. Together these two features can account almost fully for the decline in the average job-to-job transition rate. Thus just analysing the aggregate trends one may conclude, that it is not important to account for heterogeneity across workers and jobs. However, we now turn to the dis-aggregated effects by worker type. Similar to the effects on employment, the aggregate results are hiding substantial heterogeneity by education level.

Figure 9: Change in Job-to-Job Transition Rates by Education

(a) Labor Supply and Demand

(b) Search Costs and Match Effects

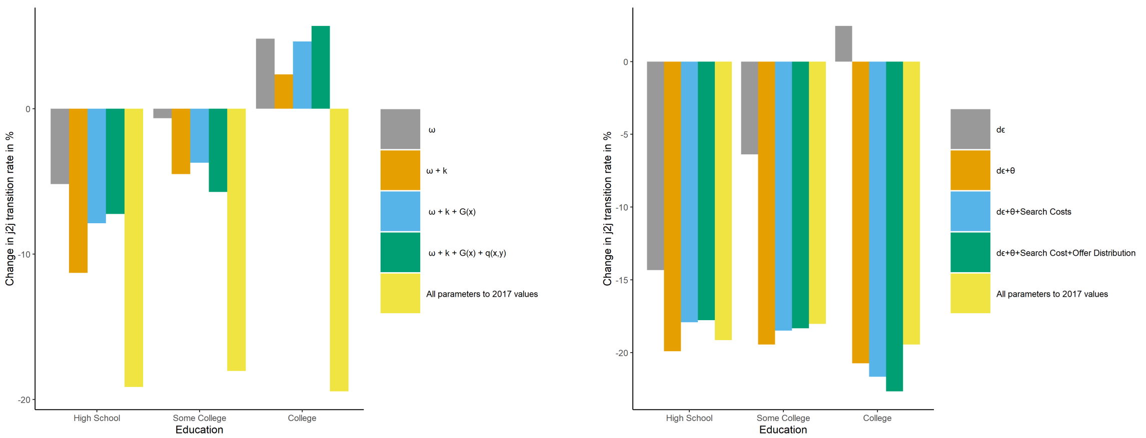


Figure 9 shows the change in job-to-job transition rates by education level, separately in response to labor supply and demand shifts in 9a and in response to search costs and match effects in 9b. Panel 9a shows the change in job-to-job transition rates by education level in response to shifts in the main labor demand and supply determinants. The shifts in labor demand driven by the occupation productivity ω_y and entry costs k_y have a negative effect on job-to-job transition rates for workers with at most some college education, while increasing mobility for workers with college education. The differences in those effects follow from the strong sorting patterns of workers to jobs, which imply different exposure to the changes in labor demand with college educated workers most likely to benefit from increased demand for non-routine cognitive jobs. Further, the very large share of job-to-job transitions within occupation groups further intensifies the differential exposure to demand shifts by education level. Taken together the overall shift in the main labor supply and demand determinants, as shown by the fourth bar in Panel 9a, can account for 35% and 32% of the decline job-to-job transition rates for workers with at most a high school degree and workers with some college education respectively. However, for college educated workers the counterfactual implies increased job-to-job transition rates. Thus, the results are consistent with a view of the labor market that implies for workers without a college degree increased competition and lower chances for mobility between jobs. Still, job-to-job transition rates have also declined for workers with college education. Panel 9b shows the results in response to changes in the search costs and match effects distribution. The results imply that the decline in the arrival rate of shocks had a larger negative impact for workers with college education and thus implies overall similar declines in job-to-job transition rates.

The results indicate, that it is important to account for both labor supply and demand shifts in order to explain the observed changes in employment patterns across education and occupation groups. Further, the impact of the shifts in labor demand and supply have heterogeneous effects on workers depending on their education level. Workers without a college degree find it harder over time to move to new employers in face of less demand for typical jobs up the job ladder, and in face of increased competition by college graduates.

6 Conclusion

A large literature has documented job polarization and recently more and more research points toward declining worker mobility. My analysis suggests that these two phenomena are linked. To study the phenomena, I propose a theoretical framework of the labor market with two-sided heterogeneity, search frictions, and on-the-job search where the demand for occupations is endogenous. Further, in line with recent evidence and to allow for realistic job-finding rates across education and occupation groups, I account for employer ranking of applicants. I apply this framework to study the recent trends in employment and the decline in job-to-job mobility. I find that routine-biased technological change not only gives rise to job polarization but also shortens the job ladders of workers without a college degree. With shorter job ladders, workers move less often between jobs, and therefore mobility declines. The shifts in demand and supply of labor across jobs can account for almost 35% of the total decline in job-to-job mobility for workers without a college degree. For workers with a college degree changes in the arrival rate and dispersion of match-specific productivity have a relatively larger effect. The results indicate, that to understand recent trends in the labor market it is important to consider underlying changes in the demand for and supply of skills. Those shifts matter above and beyond composition. It is their equilibrium interactions that are important to understand the observed trends in employment allocations by education level and occupation, and the decline in job-to-job transition rates.

The framework presented in the paper, or parts thereof, has many possible applications as it provides an appealing way to take into account the role of two-sided heterogeneity and sorting for labor market outcomes. In particular, the ranking of applicants by firms allows to capture directly the competition between heterogeneous workers for jobs. One application of the framework is to study sorting based on (un)observed heterogeneity using matched employer-employee data, which allows a nuanced view of the job search process. For example, Card, Heining, and Kline (2013) indicates that sorting based on unobserved heterogeneity has contributed to rising wage inequality in Germany. The interpretation of the common two-way fixed effects approach to estimate unobserved heterogeneity from wages pioneered by Abowd, Kramarz, and Margolis (1999) in terms of sorting is however difficult. The model presented here can be used to interpret the findings from wage regressions *jointly* with worker mobility. A step in that direction was taken by Abowd, Kramarz, Pérez-Duarte, and Schmutte (2018) who apply the static assignment model of Shimer (2005a) directly to US administrative data. However, as a high share of overall hires is directly from other employers it is important to explicitly account for such transitions in the theoretical framework. Further, one important insight of allowing firms to rank applicants in a non-sequential hiring process, is that competition by other workers would be interpreted as a job amenity through the lens of a model with a fully sequential hiring process. The micro-foundations of the job search and hiring process are thus important for the interpretation of job amenity estimates, as in for example Taber and Vejlin (2020) or Sorkin (2018).

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A Derivations

A.1 Value Functions

Surplus. Define the value of unemployment and a vacant job

$$rU(x) = \max_s b(x) + s\Omega(x, 0) - c(s) \tag{20}$$

$$rV(y) = -k(y) + \sum_{x,o} \mu_y(x, S_o) \sum_{\epsilon} f(\epsilon) \max\{J(x, S_o, \epsilon, y), 0\} \tag{21}$$

A contract specifies a sequence of payments w depending on the worker type x, job type y, match specific productivity ϵ and the outside option of the worker S_o when the worker was hired. Additionally the contract specifies a transfer $P(x, S_o, \epsilon, y)$ between worker and job in case the worker makes a job-to-job move.

$$\begin{aligned}
rW(x, S_o, \epsilon, y) =& \max_s w(x, S_o, \epsilon, y) + \theta_y \sum_{\epsilon'} f(\epsilon') \max W(x, S_o, \epsilon', y) - U, 0 - \delta(x, y)(W - U) \dots \\
&+ s\Omega(x, E + P - U) - c(s) \\
rJ(x, S_o, \epsilon, y) =& p_y \epsilon q(x, y) - w(x, S_o, \epsilon, y) + \theta_y \sum_{\epsilon'} f(\epsilon') \max J(x, S_o, \epsilon', y), 0 - \delta(x, y)(J - V) \dots \\
&- \sum_{y}^{\prime} P(\text{apply to y'}|x, S_o) \nu_{y'}(x, W + P - U)[\min\{W' - W, J\} + P(x, S_o, \epsilon', y)].
\end{aligned}$$

$P(\text{apply to y'}|x, S_o)$ is defined in equation (2).

I assume that contracts are complete and can be enforced. Therefore, the contract will maximize the total value of the match, as any contract that does not is Pareto dominated.

Now, I specify a penalty schedule $P(x, S_o, \epsilon, y)$, that maximizes total match value. To that end separate the set of alternative jobs $\{y', \epsilon'\}$ into two non-overlapping sets: (1) jobs whose total value is lower than that of the current match, (2) jobs whose total value is at least as large as that of the current match. For jobs in the first set, any application presents a net loss in terms of total private value, because the maximum possible amount of contract value offered to the worker can not compensate for the loss of value for the job owner. For jobs in the second set, applications are valuable because the worker will be able to compensate the job owner for his losses. Specify the penalty for a job-to-job move as $P(x, S_o, \epsilon, y) = [\min\{W(x, S_o, \epsilon', y') - W(x, S_o, \epsilon, y), J(x, S_o, \epsilon, y)\}$. It follows that applications to jobs which offer $W' \leq W + J$ offer no value to the worker. Therefore, she will only apply to jobs which offer $W' > W + J$. Further denote the difference of a value function compared to its current value as $\Delta J(x, S_o, \epsilon', y) = J(x, S_o, \epsilon', y) - J(x, S_o, \epsilon, y)$. Note that in equilibrium the value of a vacancy is $V(y) = 0$. It follows, that

$$\begin{aligned}
rJ(x, S_o, \epsilon, y) =& p_y \epsilon q(x, y) - w(x, S_o, \epsilon, y) + \theta_y \sum_{\epsilon'} f(\epsilon') \Delta J(x, S_o, \epsilon', y) - \delta(x, y)(J - V) \\
r[W(x, S_o, \epsilon, y) + J(x, S_o, \epsilon, y)] =& p_y \epsilon f(x, y) + \theta_y \sum_{\epsilon'} f(\epsilon') \Delta J(x, S_o, \epsilon', y) + \Delta W(x, S_o, \epsilon', y) - \delta(x, y)(W + J - U) \dots \\
& - c(s^*) + s^* \Omega(x, J + W - U) \\
S(x, \epsilon, y) =& W(x, S_o, \epsilon, y) + J(x, S_o, \epsilon, y) - U(x) \\
rS(x, \epsilon, y) =& p_y \epsilon q(x, y) + \theta_y \sum_{\epsilon'} \Delta S(x, \epsilon', y) + \max_s s\Omega(x, S) - c(s) \dots \\
& - \delta(x, y) S(x, \epsilon, y) - b(x) - s^U \Omega(x, 0) + c(s^U)
\end{aligned}$$

Surplus is independent of the current surplus split, because the gain from on-the-job search to the worker is only whatever the new job offers above and beyond the total match value of the current match. This is achieved by setting the penalty for a job-to-job move equal to the loss for the job owner. This contract maximizes surplus value, because the worker already maximizes his private value and the jobs valuation of the match is independent of on-the-job search as the job owner is compensated for any loss.

A.2 Proposition 1

Proposition 1 describes a case when there is no sorting in the economy.

Proof. To proof proposition 1, one needs to verify no sorting occurs if $q(x, y)$ is log-modular and entry costs are independent of job type $k(y) = k > 0$. For simplicity we also assume θ and match-specific productivity distribution $F(\epsilon)$ are independent of job type. Finally, the example assumes that idiosyncratic tastes for jobs are absent, but this is irrelevant to the results here.

I make a guess for equilibrium surplus and verify the resulting allocation satisfies the equilibrium conditions and implies no sorting.

Guess that surplus is independent of job-type, that is $S(x, \epsilon, y) = S(x, \epsilon, y') = S(x, \epsilon)\, \forall y, y' = 1, \dots, Y$.

The free entry condition is

$$k = \sum_{x, S_o, \epsilon} \mu_y(x, S_o, \epsilon) \widehat{S}(x, S_o, \epsilon) - \lambda_y(x, S_o) \Omega(x, S_o)$$

As surplus is the same across jobs, it follows that also the job filling rate (and therefore queue length λ) are independent of job type, i.e. $\mu_y(x, S_o, \epsilon) = \mu(x, S_o, \epsilon)$ and $\lambda_y(x, S_o, \epsilon) = \lambda(x, S_o, \epsilon)$.

Optimal contract posting (7) and worker indifference (18),

$$\Omega(x, S_o) \geq e^{-\lambda(x,S_o)} e^{-\Lambda(x,S_o)} E_\epsilon \left[\widehat{S}(x, S_o, \epsilon)\right] \\ - \sum_{x',o'} e^{-\Lambda(x',S_{o'})}(1 - e^{-\lambda(x',S_{o'})}) \mathbf{1}\{E_{\epsilon'}[\widehat{S}'] < E_\epsilon[\widehat{S}]\} E_\epsilon[\widehat{S}(x', S_{o'}, \epsilon')] \tag{22}$$

$$\lambda(x, S_o) \geq 0, \tag{23}$$

then imply that the same contract values are posted for all jobs y and that all jobs have the same job finding rate per unit of search effort. From the results in Shimer (2005a) we know the just shown first order condition has a unique solution given surplus.

Next, denote the number of matches created of a type $m(x, S_o, y)$ and following the definition of μ it holds that

$$m(x, S_o, y) = \mu_y(x, S_o) v_y.$$

Thus the ratio of matches created across jobs

$$\frac{m(x, S_o, y)}{m(x, S_o, y')} = \frac{v_y}{v_{y'}},$$

is the same as the ratio of vacancies, because job filling rates are the same. Thus the distribution of inflow into employment is independent of worker type. There is no sorting in hiring. Then sorting could still occur if for example low skill workers are more likely to separate from high skill jobs than high skill jobs or vice versa. See equation (**??**). However, as $S(x, y, \epsilon) = S(x, y', \epsilon)$ it directly follows that separation rates are independent of job type y, that is both on-the-job search and separations to unemployment are independent of y. With an abuse of notation let me denote the total flow of separations from a job by $\delta(x, \epsilon)e(x, y, \epsilon)$. That is workers of different types might separate at different rates from jobs, but they do so independent of a jobs type y. Employment across job types follows from

$$\begin{aligned} 0 &= \sum_{S_o} m(x, S_o, y) - \delta(x, \epsilon)e(x, y, \epsilon) \\ \sum_{S_o} m(x, S_o, y) &= delta(x, \epsilon)e(x, y, \epsilon) \\ \frac{\sum_{S_o} m(x, S_o, y)}{\sum_{S_o} m(x, S_o, y')} &= \frac{e(x, y, \epsilon)}{e(x, y', \epsilon)} \\ \frac{v_y}{v_{y'}} &= \frac{e(x, y, \epsilon)}{e(x, y', \epsilon)}, \end{aligned}$$

where I use that employment inflows are proportional to vacancies across job types. From the last line it directly follows that employment can be written as $e(x, y, \epsilon) = e_1(x, \epsilon)e_2(y)$, that is it is log-modular.

Job production was assumed to be log-modular in worker and job types, i.e. $q(x, y) = q_1(x)q_2(y)$.

Output in each occupation follows

$$Q_y = e_2(y)q_2(y)\sum_{x,\epsilon} e_1(x,\epsilon)q_1(x).$$

Thus, the relative price of output across job types follows

$$\frac{p_y}{p_{y'}} = \left(\frac{e_2(y)q_2(y)\omega_{y'}}{e_2(y')q_2(y')\omega_y}\right)^{-\frac{1}{\sigma}}, \tag{24}$$

which pins down the value of v_y. Search effort satisfies the definition of λ_y and market clearing $\sum_y \lambda_y(x,S_o)v_y = s(x,S_o)h(x,S_o)$. Further, the guess in terms of surplus being independent of job type y does not violate its definition. Thus all equilibrium conditions are satisfied.

Finally, we have already verified that no sorting occurs, as the employment distribution is log-modular $e(x,y,\epsilon) = e_1(x,\epsilon)e_2(y)$. □

B Examples: Parameters

The examples in section 3.1 and 3.2 use the following parameters, apart from those explicitly specified in the main text.

Table 8: Parameters for example allocation

Parameter	Value
ω_y	1
ϕ	1
ϕ - employment	0.05
σ	0.9
$k(y)$	0.5
$G(x)$	$1/N_x$

C Standard Errors

Following Gourieroux et al. (1993)

$$\hat{\theta} = \arg\min_{\theta} \sum_{i} \omega_i \left(\bar{m}_i - m_i(\theta)\right)^2 , \tag{25}$$

the variance covariance matrix of the estimates $\hat{V}$ is

$$\hat{V}(\theta) = (\hat{M}'\Omega\hat{M})^{-1}\hat{M}'\Omega\hat{\Sigma}\Omega\hat{M}(\hat{M}'\Omega\hat{M})^{-1} \tag{26}$$

where $\hat{\Sigma}$ is the variance covariance matrix of the moments m_i. $\hat{M}$ is the jacobian of the moments with respect to the parameters. And Ω is the weight matrix, here $\Omega = diag(\omega_i)$.

I set ω_i to the inverse of the estimated sample standard error, based on monthly estimates from the CPS data, while rescaling job finding rates by 5 and the standard deviation of wages by 1/5.

D Additional Descriptive Statistics

In this section I provide some additional statistics omitted from the main text.

Table 9: Additional Moments used in estimation

(a) Education shares in labor force in %

Education	Data 97	Data 17	Δ Data
High School	43.0	32.1	-11.0
Some College	28.8	27.7	-1.03
College	28.2	40.2	12.0

(b) Employment to Unemployment Transition Rate in %

Education	Occupation	Data 97	Data 17	Δ Data
High School	non-routine manual	2.3	2.0	-0.32
	routine manual	2.2	2.1	-0.13
	routine cognitive	1.2	1.4	0.2
	non-routine cognitive	0.93	1.1	0.19
Some College	non-routine manual	1.5	1.4	-0.09
	routine manual	1.6	1.7	0.11
	routine cognitive	1.0	1.1	0.083
	non-routine cognitive	0.72	0.96	0.24
College	non-routine manual	1.3	1.1	-0.26
	routine manual	1.3	1.3	-0.072
	routine cognitive	0.82	0.82	0.0053
	non-routine cognitive	0.55	0.63	0.084

Notes: Data moments based on own calculations using CPS Basic Monthly Files.